**Chapter 7**

**207 Chapter 7 “Properties of Functions” Proof Questions**

**#1.**

A function, *F*: **R** *×* **R** *→* **R** *×* **R** has been defined as follows: **F(x, y) = (3y − 1, 1 − x)** for all **(*x, y*) in R *×* R**. Prove that F is a one-to-one correspondence that is, F is both One-to-one and Onto.

**#2.**

Let S be the set of all even integers, and define a function, **f:** **Z** → **S** as follows: **f(n) = 2n** for all integers n. Prove that f is one-to-one and onto.

**All Chapters 1-10**

**207 Chapter 1 “Speaking Mathematically“ Proof Questions**

**#1.**

Define functions *F* and *G* from **R** to **R** by the following formulas:

*F*(*x*) = (*x* + 1)(*x −* 3) and *G*(*x*) = (*x −* 2)2 *−* 7*.*

Prove that *F* ≠ *G*.

**#2.**

Define a relation *R* from **R** to **R** as follows: For all (*x, y*) *∈* **R***×***R**, (*x, y*) ∈ Rif, and only if, *x* = *y*2 + 1*.* Prove that (17, -4) ∈ R

**207 Chapter 2 “Logic of Compound Statements” Proof Questions**

**#1.**

Given the following premises:

(1) p

(2) (p∨r)→s

(3) ~s∨q

Prove q.

**#2.**

Given the following premises:

(1)~p∨s

(2)s→~r

(3)r∨q

Prove p→q using a conditional world proof.

**207 Chapter 3 “The Logic of Quantified Statements” Proof Questions**

**#1.**

Prove that the following two statements are not logically equivalent. In your proof, completely justify your answer.

(a) A real number is less than 1 only if its reciprocal is greater than 1.

(b) Having a reciprocal greater than 1 is a sufficient condition for a real number to be less than 1.

**#2.**

Prove that the following is a valid argument:

All real numbers have nonnegative squares.

The number *i* has a negative square.

Therefore, the number *i* is not a real number.

**207 Chapter 4 “Methods of Proof” Proof Questions**

**#1.**

Disprove the following statement. Then write the negation of the following statement, and prove that the negation of the statement is true.

***∀* integers *m* and *n*, if 2*m* + *n* is odd then *m* and *n* are both odd.**

**#2.**

Prove that the following statement is false: The product of any two irrational numbers is irrational.

**207 Chapter 5 “Sequences, Induction, Recursion” Proof Questions**

**#1.**

Use mathematical induction I to prove that for all integers *n ≥* 3*,*

2 *·* 3 + 3 *·* 4 + *· · ·* + (*n −* 1) *· n* = (*n −* 2)(*n*2 + 2*n* + 3)/3*.*

**#2.**

Use mathematical induction II to prove that for all integers *n ≥* 5, 1 + 4*n <* 2*n.*

**207 Chapter 6 “Set Theory” Proof Questions**

**#1.**

Sets A and B are defined as follows: A = {n ∈ **Z** | n = 8r − 3 for some integer r} and B = {m ∈ **Z** | m = 4s + 1 for some integer s}.

(a) Prove that A ⊆ B.

(b) Disprove that B ⊆ A.

**#2.**

**Prove that for all sets A and B, (A − B) ∩ B = ∅.**

**207 Chapter 7 “Properties of Functions” Proof Questions**

**#1.**

A function, *F*: **R** *×* **R** *→* **R** *×* **R** has been defined as follows: **F(x, y) = (3y − 1, 1 − x)** for all **(*x, y*) in R *×* R**. Prove that F is a one-to-one correspondence that is, F is both One-to-one and Onto.

**#2.**

Let S be the set of all even integers, and define a function, **f:** **Z** → **S** as follows: **f(n) = 2n** for all integers n. Prove that f is one-to-one and onto.

**207 Chapter 8 “Properties of Relations” Proof Questions**

**#1.**

Let R be the relation defined on the set of all integers **Z** as follows: for all integers m and n, m R n ⇐⇒ m − n is divisible by 5. Prove that R is Equivalence Relation.

**#2.**

Let ***S***be the set of all strings of 0’s and 1’s of length 3. Define a relation ***R* on *S***as follows: ***s R t ⇐⇒***for all strings *s* and *t* in *S,* the two left-most characters of *s* are the same as the two left-most characters of *t.* Prove that ***R* is an equivalence relation on *S****.*

**207 Chapter 9 “Counting and Probability” Proof Questions**

**#1.**

Prove for all integers *n*, *k*, and *r* with *n ≥ k ≥ r* that **nCk×kCr = nCr×(n-r)C(k-r)**

**#2.**

The binomial theorem states that for any real numbers *a* and *b*,

(*a* + *b*)*n* = for any integer *n ≥* 0*.*

Use this theorem to show that for any integer *n ≥* 0*,*  = 1.

**207 Chapter 10 “Graphs and Trees” Proof Questions**

**#1.**

Prove that having *n* vertices, where *n* is a positive integer, is an invariant for **graph isomorphism**.

**#2.**

Prove that the sum of the degrees of the vertices of any finite graph is even.

**#3.**

Show that every simple finite graph has two vertices of the same degree.